

Institution: University of Liverpool

# Unit of Assessment: 10 – Mathematical Sciences

Title of case study: Metamaterial systems and routing of elastic waves in engineered structures

## 1. Summary of the impact (indicative maximum 100 words)

It is well-known that certain bridges are susceptible to potentially dangerous uncontrolled vibrations; recent examples include London's Millennium Bridge and the Volga Bridge in Volgograd. Correcting such problems after the construction of the bridge can be extremely expensive and time-consuming. Research in the Department of Mathematical Sciences at the University of Liverpool has led to a novel approach for predicting such behaviour in advance and then modifying the bridge design so as to avoid it. During the period 2011-12 this research has been incorporated into standard design procedures by industrial companies involved in bridge design. There is an economic impact for the companies concerned (avoiding costly repairs after bridge construction) and a societal impact (improvements in public safety and also avoiding the inconvenience of long-term closure of crucial transport links).

The research is based on a novel, highly non-trivial approach that has been developed to study properties of elastic waves in complex engineered structures with a multi-scale pattern. The work has been taken up by the industrial construction company ICOSTRADE S.R.L. Italy, whose design engineer Dr Gian Felice Giaccu integrated the innovative research ideas into their standard design procedures for complex structures such as multiply supported bridges. Novel designs of wave by-pass systems developed by the Liverpool group have also been embedded in standard algorithms by the industrial software company ENGINSOFT, in the framework of a project led by their project manager Mr. Giovanni Borzi.

#### 2. Underpinning research (indicative maximum 500 words)

The underpinning research was produced by the research group in Waves and Solid Mechanics at the University of Liverpool. The personnel engaged in the work includes Professors A.B Movchan, N.V. Movchan, R.C. McPhedran and Dr Michele Brun, as well as the PhD students Mr Stewart Haslinger and Mr Daniel Colquitt. The work has attracted external support in the form of an EPSRC grant [G3], large scale EU-funded Industry-Academia Partnerships and Pathways grants [G1, G2] involving industrial partners in Italy, Poland, Israel and Ukraine, the Duncan Norman Memorial Fellowship, and the EU Commission Marie Curie grant [G4]. It should be emphasised that in line with our Impact strategy, one of the end-users, Dr Giacci, has been closely involved with the research (and indeed is a co-author on one of the research papers). This has enabled us to tailor the research closely to the end-users' requirements but also implies that the research and impact are closely intertwined. For completeness, but at the risk of some repetition later, we shall describe the whole process here.

The concepts of waveguides and wave bypass structures are useful in a wide variety of contexts, ranging from natural systems which have biologically useful visual properties, to the optical fibre technologies which power the internet. These powerful ideas have not yet been effectively implemented in other areas of wave science. In particular, they can prove very effective in providing simple analytic models and results for the vibrations occurring in large multiply supported structures such as bridges, but seem to have been largely overlooked in this context; an omission which has now been remedied by our research.

An important simplifying concept which has proved extremely successful is to replace the large multiply supported structure by a single unit cell of an appropriate periodic structure, as described in [1]. This replacement permits the use of techniques familiar in Solid State Physics, known as Bloch Mode Analysis. These Bloch Modes give the allowed vibration patterns in the building blocks (unit cells) of periodic structures. There are well developed techniques for finding these Bloch Modes, and also for detecting whether such vibration modes have been overlooked. Addition of a



high contrast wave by-pass system leads to new dispersion properties of Bloch waves which are exploited to divert wave energy away from the bridge deck. Our background work on the analysis of Bloch waves in complex periodic systems is described in Refs. [1-3] listed below. Of particular relevance here is the localisation in multi-scale engineered systems studied in Refs. [2, 3].

These ideas have been developed into a method of analysing large multiply supported bridge structures in order to identify and avoid unwanted vibrations. This method does not replace the results of the complex industrial design packages which are currently used to provide final designs. However, it does provide a realistic appreciation of the characteristics and frequency ranges of vibrational modes which are likely to prove troublesome to the reliable performance of the bridge over a wide variety of environmental conditions. Importantly, it also provides an immediate indication as to whether any important vibrational modes have been overlooked in the voluminous results provided by commercial design packages. The knowledge of these troublesome frequencies is then used in the design of a lightweight "wave bypass" structure that diverts the vibrations away from load-bearing elements. Further "dumping" of unwanted vibrational modes is applied as appropriate. The bypass structure represents a highly directive system that re-routes the waves around the bridge deck, which is then shielded from vibrations within the unwanted frequency range. The design involves considering the deck of a bridge as a slender solid lying on pillars placed at regularly spaced intervals. By analysing the vibration of each repeating element or "unit cell" of the bridge, deflection of the unwanted modes away from the bridge deck is achieved by adding a system of linked resonators. The advantage of this approach is that the total mass of each resonator is several orders of magnitude less than the bridge itself, while the bars linking the resonators will have a relatively low stiffness. Such structures are easily predesigned by evaluating their frequencies of vibration when they are isolated from the bridge. A crucial feature is that this design does not require any change in the way the main deck is attached to the supporting pillars.

## 3. References to the research (indicative maximum of six references)

Research publications:

- 1. Brun M., Giaccu G.F., Movchan, A.B. & Movchan, N.V. (2012) Asymptotics of eigenfrequencies in the dynamic response of elongated multi-structures. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **468 (2138)**, 378–394. doi:10.1098/rspa.2011.0415
- Colquitt, D. J., Jones, I. S., Movchan, N. V., & Movchan, A. B. (2011). Dispersion and localization of elastic waves in materials with microstructure. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **467(2134)**, 2874–2895. doi:10.1098/rspa.2011.0126
- 3. Colquitt, D.J., Jones, I.S., Movchan, N.V., Movchan, A.B., Brun, M. and McPhedran, R.C. (2013) Making waves round a structured cloak: lattices, negative refraction and fringes. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **469** (2157), 20130218. doi: 10.1098/rspa.2013.021.

The journal *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* has an Impact Factor of 2.34 and is ranked 11 out of 56 journals in its sector by Web of Knowledge.

External grant support:

G1. European Commission FP7 Grant. Industry-Academia Partnerships and Pathways. PIAP-GA-284544-PARM-2, 1.78M Euro, Liverpool share £180,000.

G2. European Commission FP7 Grant. Industry-Academia Partnerships and Pathways. PIAP-GA-



286110-INTERCER2, 2.34M Euro, Liverpool share £160,000.

G3. EPSRC research grant EP/D035082/1 of £144,744 on "Thermal vibrations and localization for solids with singularly perturbed boundaries".

G4. European Commission Marie Curie grant on Modelling of smart composite interfaces and dynamically resistant systems, PIEF-GA-2011-302357-DYNAMETA, £220,257.

### **4. Details of the impact** (indicative maximum 750 words)

Most modern 21st-century structures are designed so that damaging low-frequency elastic vibrations can be avoided. But unexpected external loads can still trigger unwanted shake and rattle, with some dangerous consequences. Well-known examples are the Millennium Bridge in London and the Volga Bridge in the Russian city of Volgograd. The first of these opened as a footbridge across the Thames in 2000 but had to be shut soon after for a major redesign after members of the public complained about it moving excessively when they walked across it. The 7.1km road bridge over the river Volga had similar problems when a resonant vibration caused sections of the bridge to shake in May 2011, less than a year after it had opened. The fact that problems arose, even though the Volga and Millennium bridges were designed using fully certified packages, shows that it is all too easy with large and complicated structures to overlook vibrations that may cause structural problems under practical conditions [5.1].

During 2011-12, our proposed approach and the results of our novel asymptotic analysis have been fully adopted by the design engineer, Dr Gian Felice Giaccu and the design and construction company ICOSTRADE S.R.L. in their working practices, combined with the industrial grade FEM package. The use of this novel approach has made the design of elongated structures more efficient and reliable by helping to avoid unwanted low-frequency vibrations of the bridge systems, and possible design errors similar to the ones that occurred in the Millennium Bridge and the Volga Bridge. The financial implications of correcting a design error in a bridge are substantial, as multimillion investments are required on every occasion when such errors occur, and the new method has already provided a very efficient practical tool. The company has written a letter confirming the material benefits of this research on their practical operations. To quote from the letter of support [5.2]: "This new method has now been adopted in our working practices, and we acknowledge a high positive impact of this research on our business. The recent novel idea of phase transition waves, originated from this research on long bridges, we also recognized as a realistic mechanism of potential failure of bridges and hence has been taken into account in the definition of the safety margins for our industrial designs."

The approach leading to routing of elastic waves around unwanted regions has also delivered new multi-scale designs, adopted by the international software development company ENGINSOFT, which specialises in industrial computation and design. The work at ENGINSOFT was led by the project manager, Mr Giovanni Borzi, and the algorithms are linked to structured wave shields around stress concentrators such as voids or entrant corners, as well as problems of thermal striping in structured solids. This work was done in the framework of EU-funded grant [G2], valued at 2.34M Euro, that develops Industry-Academia Partnerships and Pathways. The value of the impact is high, as it has led to enhanced hybrid computational algorithms of ENGINSOFT for analysis for dynamic response of multi-scale structural systems with defects. This has advanced the capability of the company, which has a multi-million turnover of industrial research projects. A letter [5.3] has been provided by ENGINSOFT to confirm the significant impact on their industrial work made by the research of A.B. Movchan and his group. They say that Prof Movchan's new methods have "offered revolutionary new perspectives" which "have proved to be extremely effective". They confirm that these new methods "have been incorporated into our working practices and boosted our competitive position" with "an immediate financial benefit already estimated at around 250,000 Euros."

Efficient knowledge exchange events have been put in place to make sure that the results and ideas of design for wave by-pass systems for structured solids were exposed to the academic and industrial communities. These include the International Workshop "Elasticity Day", held in Liverpool



in May 2012, and three industrial workshops, all organized and held in Liverpool, on Modelling of Defects in Welds and Heterogeneous Media (May 2008), Waves in Structured Media and Localisation (June 2009) and Asymptotic & Computational Models of Fracture and Wave Propagation in Structured Media (May 2010) as well as an interdisciplinary workshop on Metamaterial Structures and Dynamic Localisation Effects (December 2011). This knowledge exchange has been also enhanced by the recent publication of the article "Bypassing Shake Rattle and Roll" [5.1] in Physics World [May, 2013]

5. Sources to corroborate the impact (indicative maximum of 10 references)

- 5.1 Bypassing shake, rattle and roll, Feature Article by M. Brun, A. Movchan, I. Jones and R. McPhedran, Physics World, May 2013.
- 5.2 The Design Engineer at ICOSTRADE S.R.L. and colleagues were primary users of the new findings for by-pass metamaterial systems in the design of bridges and routing low frequency vibrations away from the main deck and corroborate this in a statement of support.
- 5.3 The Project Manager at ENGINSOFT, can corroborate, in a letter of support that "the research results of the Liverpool group have made a significant impact on the development of novel hybrid algorithms..."